	P.R.Government College (Autonomous) **KAKINADA**	**Program&Semester** III B.Sc. (VSem)			
CourseCode **MAT-701A / 5281**	**TITLEOFTHECOURSE** **VII A -Mathematical Special Functions**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on Integration	4	1	-	4

Course Objectives:

This course will cover the particular mathematical functions that have more or less established names and notations due to their importance in mathematical analysis, functional analysis, geometry, physics, or other applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations.
CO2	Find power series solutions of ordinary differential equations.
CO3	Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials.
CO4	Solve Bessel equation and write the Bessel equation of first kind of order n, also find the generating function for Bessel function understand the orthogonal properties of Bessel unction.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

II. Syllabus: (Hours: Teaching: 75 (incl. unit tests etc. 05), Training: 15)

Unit – 1: Beta and Gamma functions.

1. Euler's Integrals-Bet and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.

2. Another form of Beta Function, Relation between Beta and Gamma Functions.

Unit– 2: Power series and Power series solutions of ordinary differential equations

1. Introduction, summary of useful results, power series, radius of convergence, theorems on Power series
2. Introduction of power series solutions of ordinary differential equation
3. Ordinary and singular points, regular and irregular singular points, power series solution about the ordinary point $x = x_0$.

Unit – 3: Hermite polynomials

1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials.
2. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials.
3. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

Unit – 4: Legendre polynomials

1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n, generating function of Legendre polynomials. ,
2. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required)to show that $P_n(x)$ is the coefficient of h^n , in the expansion of $(1 - 2xh + h^2)^{-1/2}$
3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

Unit – 5: Bessel's equation

1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n, Bessel's function of the second kind of order n.
2. Integration of Bessel's equation in series form=0, Definitionof $J_n(x)$, recurrence formulae for $J_n(x)$. 3. Generating function for $J_n(x)$, orthogonally of Bessel functions.

Additional Inputs :

Chebyshev Polynomials.

II. Reference Books:

1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
2. J.N.Sharma and Dr.R.K.Gupta, Differential equations with special functions, Krishna Prakashan Mandir. 3. Shanti Narayan and Dr.P.K.Mittal, Integral Calculus, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
4. George F.Simmons, Differential Equations with Applications and Historical Notes, Tata McGRAW-Hill Edition, 1994.
5. Shepley L.Ross, Differential equations, Second Edition, John Willy & sons, New York, 1974.
6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities:

A) Mandatory: 1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).

1. Beta and Gamma functions.
2. Power series, power series solutions of ordinary differential equations,
3. Procedures of finding series solutions of Hermite equation, Legendre equation and Bessel equation.

4. Procedures of finding generating functions for Hermite polynomials, Legendre Polynomials and Bessel's function.

2. For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work, make observations and conclusions and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.

1. Going through the web sources like Open Educational Resources on the properties of Beta and Gamma functions, Chebyshev polynomials, power series solutions of ordinary differential equations. (or)

2. Going through the web sources like Open Educational Resources on the properties of series solutions of Hermite equation, Legendre equation and Bessel equation.

3. Max. Marks for Fieldwork/Project work Report: 05.

4. Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

b) Suggested Co-Curricular Activities:

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
3. Invited lectures and presentations on related topics by experts in the specified area.

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	1	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN,

Skill Enhancement Course (Elective): VII - A

Paper – VII –A :Mathematical Special Functions

UNIT	TOPIC	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted
I	Beta and Gamma functions,	02	02	30
II	Power series and Power series solutions of ordinary differential equations	02	01	20
III	Hermite polynomials	01	01	15
IV	Legendre polynomials	01	01	15
V	Bessel’s equation	01	01	15
Total		07	06	95

S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (10 marks)

Short answer questions : 4 x 5 M = 20
Essay questions : 3 x 10 M = 30

Total Marks : = 50

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Pithapur Rajah's Government College (Autonomous), Kakinada

III Year B.Sc., Degree Examinations - V Semester

Mathematics Course VII A: Mathematical Special Functions

Paper VII A (Model Paper w.e.f. 2023-24)

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Time: 2Hrs **ax. Marks: 50**

SECTION-A

Answer Any Three Questions, Selecting At Least One Question from Each Part.

Part – A **3 X 10 = 30**

1.Essay question from Unit - I.

2. Essay question from Unit – I

3. Essay question from Unit - II.

Part – B

4. Essay question from Unit - III .

5. Essay question from Unit - IV.

6. Essay question from Unit - V .

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

7. Short answer question from Unit - I.

8. Short answer question from Unit - I .

9. Short answer question from Unit – II.

10. Short answer question from Unit - II.

11. Short answer question from Unit - III.

12. Short answer question from unit – IV.

13. Short answer question from Unit – V.

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PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA

III B.SC MATHEMATICS – Semester V

Skill Enhancement Course (Elective) :VII – A - Mathematical Special Functions

QUESTION BANK

Short Answer questions

Unit - I

1. Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$
2. Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\, dx$.
3. Evaluate $\int_0^1 \frac{dx}{\sqrt{-log_e x}}$
4. Prove that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-y^{1/n}} dy\ \ and\ hence\ show\ that\ \ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
5. Prove that if n > 0 then $\Gamma(n+1) = n\Gamma(n)$
6. Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$
7. Show that $\Gamma\left(\frac{1}{2}+x\right)\Gamma\left(\frac{1}{2}-x\right) = \frac{\pi}{\cos \pi x}$
8. Define Gamma and Beta functions. Write the relation between Gamma and Beta functions.

Unit – II

9. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for |x| < R and divergent for |x| > R .
10. Find the radius of convergence of the series $\frac{x}{2} + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \cdots$
11. Find the radius of the convergence of the series $\sum(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
12. Determine whether x = 0 is an ordinary point or a regular singular point of the differential equation $2x^2\left(\frac{d^2y}{dx^2}\right) + 7x(x+1)\frac{dy}{dx} - 3y = 0$.
13. Show that x = 0 is an ordinary point of $(x^2-1)y'' + xy' - y = 0$, but x = 1 is a regular singular point .
14. Show that x = 0 and x = -1 are singular points of $x^2(x+1)^2y'' + (x^2-1)y' + 2y = 0$ where the first is irregular and the other is regular.
15. Solve by power series method $y' - y = 0$.

Unit - III

16. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}\ and\ H_{2n+1}(0) = 0$.
17. Find Hermit Polynomials for n=0, 1, 2, 3, 4 .
18. Prove that $H_n'' = 4n(n-1)H_{n-2}$
19. Prove that $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$
20. Prove that $H_n(-x) = (-1)^n H_n(x)$.
21. Prove that, if $m < n,\ \frac{d^m}{dx^m}\{H_n(x)\} = \frac{2^m n!}{(n-m)!}H_{n-m}(x)$.

22. Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x).H_m(x)$.

Unit - IV

23. Prove that $P_n(-x) = (-1)^n P_n(x)$ *and hence deduce that* $P_n(-1) = (-1)^n$
24. Prove that $P_n' = \frac{n(n+1)}{2}$
25. Prove that $(2n+1)P_n = P'_{n+1} - P'_{n-1}$.
26. Prove that $xP_n' - P'_{n-1} = nP_n$.
27. Prove that $(n+1)P_n = P'_{n+1} - xP_n'$.
28. Prove that $(1-x^2)P_n' = n(P_{n-1} - xP_n)$.
29. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.

Unit – V

30. Prove that, when n is a positive integer $J_{-n}(x) = (-1)^n J_n(x)$.

31. Show that $J_n(-x) = (-1)^n J_n(x)$ for positive or negative integers.

32. Prove that $J_n(x) = \frac{x}{2n}[J_{n-1}(x) + J_{n+1}(x)]$

33. Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$

34. Prove that $J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\, sin\theta)\, d\theta$

35. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

36. Show that $\int_0^{\infty} e^{-ax} J_0(bx)dx = \frac{1}{\sqrt{(a^2+b^2)}}$, $a > 0$

Essay Questions
Unit –I

1. Show that $\Gamma\left(n+\frac{1}{2}\right) = \frac{1.3.5....(2n-1)\sqrt{\pi}}{2^n}$, when n is a positive integer.

2. When n is a positive integer, prove that $\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-1)^n 2^n\sqrt{\pi}}{1.3.5.....(2n-1)}$

3. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

4. Prove that $\int_0^{\pi/2} sin^{2l-1}\theta.cos^{2m-1}\theta\, d\theta = \frac{\Gamma(l)\Gamma(m)}{2\Gamma(l+m)}$.

5. Evaluate $i)\int_0^a \frac{dx}{(a^n-x^n)^{\frac{1}{n}}}$ $ii)\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx$

6. Show that $\int_0^\infty (\tan x)^n \, dx = \frac{\pi}{2} sec \frac{n\pi}{2}$, $0 < n < 1$

Unit – II

1. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for |x| < R and divergent for |x| > R .
2. Find the radius of convergence the exact interval of convergence of the power series $\sum \frac{(n+1)}{(n+2)(n+3)} x^n$
3. Determine the interval of convergence of the power series $\sum\{\frac{1}{n}(-1)^{n+1}(x-1)^n\}$
4. Determine whether x = 0 is an ordinary point or regular singular point for the differential equation $2x^2y''$ - xy' + (x-5)y = 0.
6. Show that x = 0 is an ordinary point and x = 1 is an irregular singular point of $x(x-1)^3y''$ + $2(x-1)^3y'$ +3y = 0,
5. Find the power series solution of the equation $(x^2+1)y'' + xy' - xy = 0$ in powers of x.
6. Find the solution in series of $\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right) + x^2y = 0$ about x = 0 .
7. Find the general solution of $y'' + (x-3)y' + y = 0$ near x = 2 .

Unit – III

1. State and Prove generating function of the Hermit's polynomial.
2. State and Prove Rodrigues formula for $H_n(x)$.
3. State and Prove Orthogonal Properties of Hermite Polynomials.
4. Prove that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.
5. Prove that $H_n'(x) = 2nH_{n-1}(x) \quad n \geq 1$ and $H_0'(x) = 0$.
6. Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$

Unit – IV

1. Show that $P_n(x)$ is the coefficient of h^n in the expansion of $(1-2xh+h^2)^{-1/2}$ in

   Ascending powers of h for | x | ≤ 1 and | h | < 1 .
2. Prove that $P_n(x) = \frac{1}{n!2^n} . \frac{d^n}{dx^n}(x^2-1)^n$.
3. Prove that $\int_{-1}^{1}[P_n(x)]^2 dx = \frac{2}{2n+1}$.
4. Prove that $\int_{-1}^{1} P_m(x). P_n(x) dx = 0 \ \ if \ m \neq n$. and 2/(2n+1) if m = n .
5. Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$
6. Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's Polynomials.
7. Prove that $\int_{-1}^{1}(x^2-1)P_{n+1}P_n' \, dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$.
8. Prove that $\int_{-1}^{1} x^n \, P_n(x) dx = \frac{2^{n+1}(n!)^2}{(2n+1)!}$

Unit V

1. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.

2. Prove that $xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x)$.

3. Prove that $x^2 J''_n(x) = (n^2 - n - x^2)J_n(x) + xJ_{n+1}(x)$

4. Prove that $i)\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ $(ii)\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$.

5. Prove that $\sqrt{\frac{\pi x}{2}} J_{3/2}(x) = \frac{1}{x}\sin x - \cos x$.

6. Show that $\cos x = J_0 - 2J_2 + 2J_4 - \cdots$ .... and $\sin x = 2J_1 - 2J_3 + 2J_5 - \cdots$ ...

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